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Development of envelope curve for Indus and Jhelum River basin in Pakistan and estimation of upper bound using envelope curve

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Abstract

Envelope curves developed by plotting the largest flood peaks versus the catchment area can be used in estimating peak flood discharges. This study, therefore, provides a basic understanding of the relationship between the basin area and the peak flood. Envelope curve developed for Indus and Jhelum River basins is obtained by using the recent data as well, can be used for the estimation of peak flood discharge that has occurred in the specific region within these catchments and compared with the curve for Danube River basin. This work is also helpful for the improvement of flood frequency analysis at high return periods. Purpose of this work is deriving an upper bounds and to use them in the distribution function. By this approach, the estimation of discharge for higher return periods seems to provide more realistic discharge estimation for higher return periods.

Keywords: Envelope curve; PMF; Gumbel method; Log Pearson Type- III; Two parameter gamma.

1. Introduction

Different techniques are being used for flood estimation and mitigation. Frequency analysis, Probable Maximum Flood (PMF) approach, Empirical formulae and Rational methods are the most commonly used method all over the world. All of these methods have their own limitations.

Envelope curve is also used for the estimation of flood which has occurred in a certain size of the catchment. Envelope curve obtained by plotting the largest flood peaks versus the drainage area provides an upper bound value of flood within a certain catchment. The study of largest floods observed in Pakistan is very important to calculate the maximum flood that can occur in a river basin. For this purpose, Indus and Jhelum River basins were selected for the development of envelope curves. The maximum peak discharges observed at gauging stations in the selected regions were plotted versus the basin area to obtain an envelope curve such that the entire observed flood discharges lie below this curve. The envelope curves thus obtained can be used in preliminary studies, conducted for the design of hydraulic structures. Areas which have same climatic conditions were grouped together for the development of envelope curve.

All the data available regarding discharges was collected along with their respective catchment area from Surface Water Hydrology Project (SWHP) WAPDA. Peak flood discharges observed at gauging stations present in Indus and Jhelum River basins were then plotted against drainage area and a smooth curve was drawn to cover or envelope the highest plotted points. Two different approaches were used to envelope the plotted points. In first approach, plotted points were enveloped by using creager's equation, developed for Danube River basin. In second approach, best fit line method was used to envelope the plotted points. Both of these methods were found good for the development of envelope curve in Pakistan. The method of envelope curve is a regional analysis method which assumes that there is a unique relationship between the maximum flood and basin area in a region that is hydro-climatically homogeneous.

Bayazit and Onoz (2004) have developed envelope curves for different river basins in Turkey. They stated that envelope curves obtained by plotting the largest flood peaks versus the drainage area is a good idea in the actual estimation of flood discharges, with the methods associated with flood frequency analysis as well as Probable Maximum Flood

(PMF). They developed the envelope curve for the river basins within Turkey using the results associated with DSI (state hydraulic functions administration) study using the data input together until 1990. A curve for Turkey was developed while using recent data too, and then compared with the envelope curves for the world.

John and Crippen (1982) have stated that maximum flood experienced inside a region could be described with a graph (log-log scale) on which maximum observed floods tends to be plotted against drainage area. An envelope curve covering all the plotted points has an upper bound value for the maximum observed floods. They developed envelope curves for 17 regions in the U.S.A and tend to be described through equations. These curves don't provide the actual frequency associated with flood since they're developed based on observed flood, but, inside the region to which they can apply, they provide evidence regarding the magnitude of flood which was occurred.

Matalas et al. (2007) stated that Envelope curves provide a summary of flood events occurred across a region, but their use is limited because of the inability to assign them exceedance likelihood. Analytical results are reported for the case whenever floods follow to a Gumbel or even Generalized Extreme Value distribution, and these results are contrasted along with those associated with previous research that searched for the estimation of exceedance probability associated with exceptionally large floods like the flood of record. A case study related to Flood of Records (FOR) as well as PMF discharges for 226 rivers across the United State of America, indicates that regular estimates of exceedance probability related to both PMF and FOR envelope curves can be acquired using the actual theoretical approach introduced right here.

Many empirical formulae have been developed on the basis of statistical correlation of observed flood peaks for the south Asian region. In these formulae area of the basin is the only independent variable. These formulae only provide the magnitude of flood but cannot be used to estimate flood of various frequencies which may be required as design criteria for different type of structures. Hence these formulae are not useful for determination of peak flood and its hydrograph for large projects where danger to life and property is involved. Flood frequency analysis on the other hand requires large flood records in the basin. Because the number of records is usually small, it is necessary to fit a probability distribution function to the observed flood records for estimating the flood discharge of a high return period. It is very difficult to select the best fit distribution because results of distributions vary significantly with increasing return period. Calculated discharge may vary considerably with the selected distribution. Due to short time series and rare extreme events, the results of a flood frequency analysis are uncertain especially for return periods of greater than100 years.

Figure 1 shows that differences between the results of six distribution functions are rising with increasing return period. The Probable Maximum Flood (PMF) approach can also be used for determination of peak flood. However, the PMF approach requires sufficiently detailed hydro-meteorological study. Both methods are difficult to use in basins with small data, in which case the envelope curve of maximum observed floods can be helpful in estimating the flood discharges, especially for un-gauged sub-basins. The rational method is used in cases where the catchment area is 40 acres or less. The rational method is a standard method for calculating the peak runoff rate. The results of its use are very sensitive to the coefficients selected. So this method is suitable only for small catchment areas. For larger areas, some other accurate methods should be used. In this back drop, the envelope curve can be helpful in estimating peak flood in basin of known area with comparatively less data than other approaches. Its comparison with the different frequency distributions can be useful in selecting appropriate distribution for the basin under consideration.

Envelope curve was developed for the selected regions. Flood discharges at different return periods were estimated by using different frequency distributions (Gumbel, log-Pearson type III distribution, two parameter gamma distribution and log-normal) for all stream gauging stations under study. Peak floods for un-gauged and gauged sub-basins were then estimated with the envelope curve developed for Indus and Jhelum River basins and the results were validated with conventional method (empirical formulae). Finally envelope curve developed for the selected regions was compared with the envelope curve developed for different global regions.

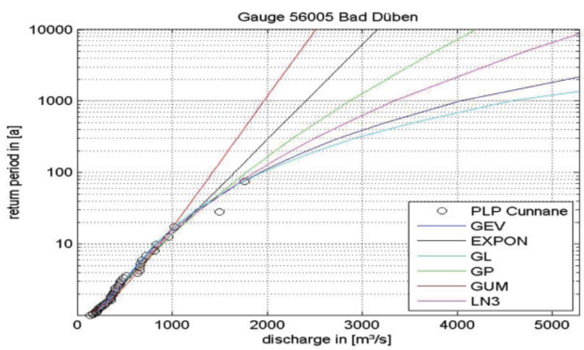


Fig 1. Variations in distributions with increasing return period (Guse, 2007).

2. Methodology

Indus and Jhelum River basins are the main river basins in Pakistan. Most of the hydraulic structures are constructed on Indus and Jhelum River basins, therefore these two river basins were selected for the development of envelope curves and detailed analyses. The maps of Indus and Jhelum River basins are shown in figure 2 and figure 3 respectively.

2.1. Data collection

The daily flow data of all stream gauging stations in the selected regions was collected for a period of at least 30 years from 1960 to 2010. The data was collected from Surface Water Hydrology Project (SWHP), WAPDA. All of the sub basins are hydro-climatically homogeneous within these regions. The details, descriptions and location about the main basins (Indus and Jhelum River basins) and their sub-basins are given in Table 1 and Table 2.

2.2. Data plotting

After collecting the data, maximum flood peaks observed at all stream gauging stations were determined by interpolation of daily flow data. In the next step, these flood peaks were plotted against the catchment area. After that a smooth curve was drawn in a way that all flood peaks lie below this curve. On the other hand, discharge values for 10, 50, 100, 1000 and 10,000 years return periods were estimated by using different frequency distributions. Four distributions (Gumbel, log-Pearson type III, Two parameter gamma and log-normal) were used for the estimation of discharges at different return periods. The frequency analysis was also performed by using Hyfran plus software for results validation. Discharge values for different return periods estimated by flood frequency analysis and Hyfran Plus were approximately same. These discharges were then plotted against return periods for each gauging site.

2.3. Development of envelope curves

For the development of envelope curve, available flood peak data from a large number of catchments (Indus and Jhelum) was collected. Then the data was plotted on a log-log graph as flood peaks versus catchment areas. This resulted in a scattered data plot. Then plotted points were enveloped by a smooth curve. The envelope curve was drawn by using two different approaches:

i. Linear relationship between peak discharge and area of sub catchment
ii. Linear relationship between peak discharge as calculated using Creager's equation versus area of sub catchment

2.3.1. Linear relationship between peak discharge and area of sub catchment

The envelope curves were developed by following the steps as:

1. Daily flow data of all stream gauging stations in the selected river basins was collected from WAPDA (SWHP).
2. Annual flood peaks of all stream gauging stations were determined from daily flow data.
3. Maximum flood peak observed at each stream gauging station was computed by

interpolation of annual flood peaks.

4. Catchment areas of all stream gauging stations were collected from reports of Surface Water Hydrology Project (WAPDA).
5. After that maximum flood peak observed at each gauging station was plotted against catchment area of the respective station on log-log graph.
6. After plotting the data a best fit line was drawn, and then drew a line parallel to this best fit line in a way that it enveloped all the points.

2.3.2 Linear relationship between peak discharges as calculated using creager equation versus area of sub catchment

Same process as of steps 1 to 6 (2.3.1) was repeated in this approach but in second approach Creamer's equation was used to envelope the points. Equations 2.1 and 2.2 show basic Creamer's equations in different forms.

$$Q=KA\gamma \qquad (2.1)$$

Where;

Q= the largest observed flood (m^3/s or ft^3/s) at a given river basin (record flood).
K= a regional statistical coefficient.
A= Catchment Area (km^2 or mi^2).
γ = an exponent less than unity,

Values assigned to γ by various investigators have ranged from 0.3 to 0.8(Castellarin, 2007).

$$Q=46CA\ 0.894\ A\text{-}0.048 \qquad (2.2)$$

Where;

Q = Peak flow in ft^3/sec
A = Drainage area in mi^2
C = Creager's coefficient

2.4. Development of Indus and Jhelum envelope curve using different creager coefficient values

Creager's equation has been used worldwide with different coefficient values. Equation 2.1 was used with different Creager's coefficient values for the development of Indus and Jhelum envelope curve. Different Creager's coefficient values were used as a trial until envelope curve covered all the flood peaks of stream gauging stations.

2.5. Estimation of upper bound using envelope curve

Envelope curves are a traditional method to appraise the upper bound of flood event. To improve flood frequency analysis for high return periods, the upper bound was derived from envelope curve for each gauging station present in Indus and Jhelum River basins and was integrated as supplementary information in a distribution function. All the selected distribution, log-normal, log-Pearson type Ш, Gumbel and two parameter gamma were applied on each gauge site.

2.6. Estimation of peak flood for certain sub-basin with the envelope curve and its comparison with empirical formulae

Different stations were selected for the estimation of peak flood by envelope curve. These stations are present in Jhelum and Indus River basins and have different catchment area. The estimated flood discharges were than compared with the flood discharges estimated by Dicken's formulae as shown below.

$$Q = C.\ A\ 3/4 \quad (2.3)$$

Where;

Q = Discharge in m^3/s
A= Area in sq.km.
C = 6 for North-Indian Plains
= 11-14 North-Indian Plains
= 14-28 Central India
= 22–28 Coastal Andhra & Orisa

2.7. Comparison of Indus Jhelum Envelope curve with Danube River basin envelope curve

The Danube River basin is the second largest basin in Europe having a catchment area of about 817000 km^2, roughly 60% of the country's land area, where approximately 65% of the total population of the Republic of Croatia. Other Croatian Rivers, such as the

Danube, the Sava, the Drava etc.; flow through this area. It is located on the Pannonian plain and its rims, with the water divide separating it from the Adriatic catchments running through the Dinaric karst. Creager and Francou-Rodier have developed envelope curves for highest observed discharges in the Danube River basin in Croatia. Creager's envelope curve was selected to compare it with the combine envelope curve of Indus and Jhelum River basin. The envelope curves for both river basins (Danub and Indus Jhelum Basin) were drawn on a log-log scale to check the behavior of both river basins. Map of Danube River basin is shown in figure 4.

Table 1 . Selected sub-basins and gauging stations in Jhelum River basin.

Main Basin	Sub-Basins	Stations	Name of gauging stations	Data Range
Jhelum	Kunhar basin	2	Naran, Garhi-Habibullah	36 year
Jhelum	Neelum basin	3	Nosheri, Muzaffarabad,Domel	46 year
Jhelum	Jhelum basin	4	Azadpattan, Chinari, Kohala,Mangla.	30 year
Jhelum	Poonch basin	1	Kotli	45 year
Jhelum	Kanshi basin	1	Palote	35 year

Table 2. Selected sub-basins and gauging stations in Indus River basin.

Main Basin	Sub-Basins	Stations	Name of gauging stations	Data Range
Indus	Gilgit	1	Alam bridge	38 year
Indus	Gomal	1	Kot murtaza	42 year
Indus	Kurram	1	Thal	39 year
Indus	Sil	1	Chahan	44 year
Indus	Soan	3	Chirah ,Dhok pathan,Gorakhpur Brigde	46 year
Indus	Haro	1	Gurriala	38 year
Indus	Kabul	1	Nowshera	46 year
Indus	Bara	1	Jhansi post	46 year
Indus	Swat	2	Kalam,Chakdara	46 year
Indus	Chitral	1	Chitral	43 year
Indus	Siran	1	Phulra	37 year
Indus	Barando	1	Daggar	37 year
Indus	Gorband	1	Karora	29 year
Indus	Astore	1	Doyian	31 year
Indus	Shyok	1	Yogo	34 year
Indus	Indus	7	Kharmong.Kachura,ShatialBridge,Besham Qila,Khairabad,Massan.Dado Moro Bridge	37 year

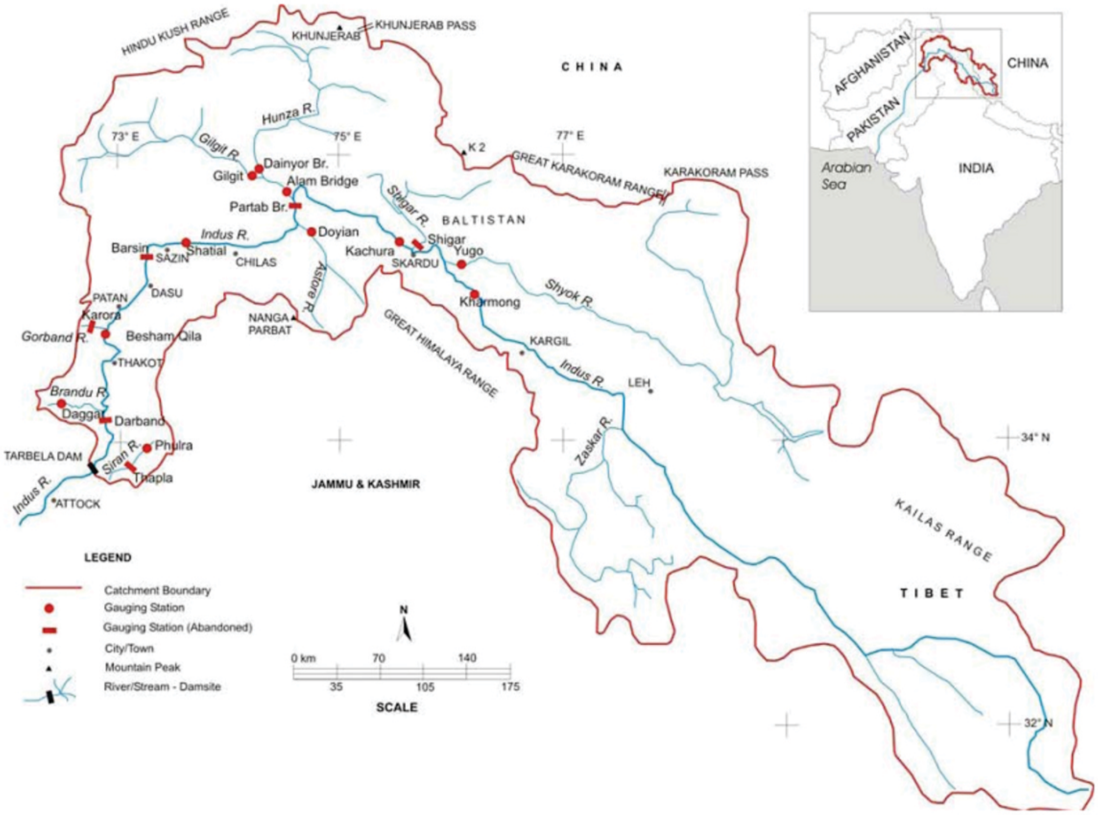
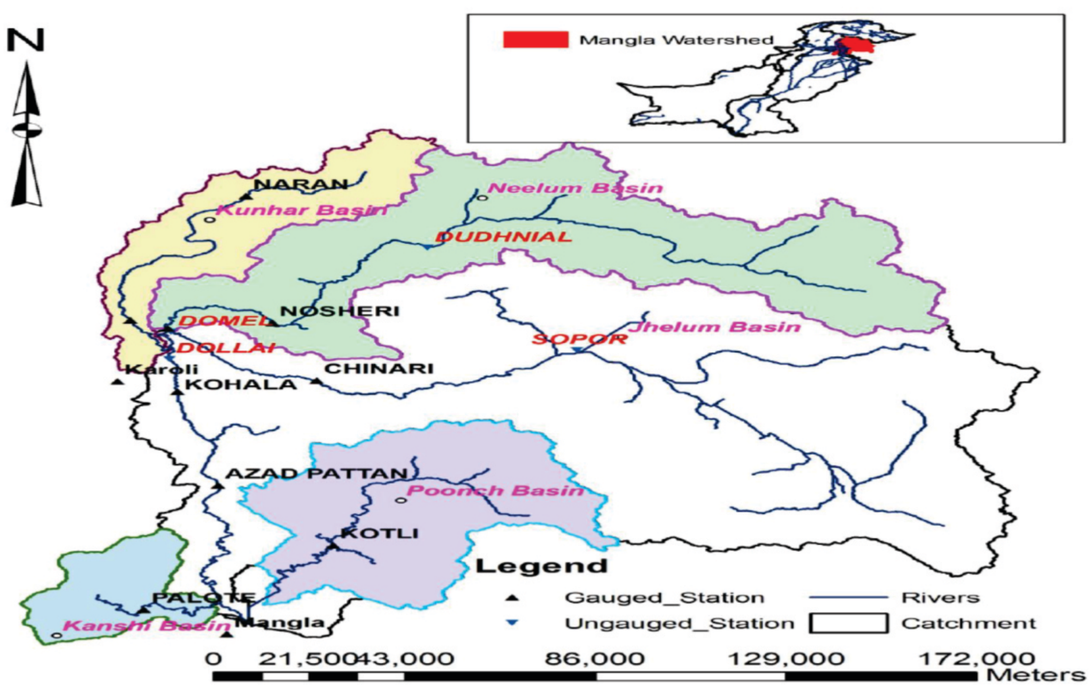


Fig. 2. WAPDA stream gauging network in upper Indus basin.



Ungauged stations
1. Domel, 2.Dollai,
3. Dhudnial, 4. Sopor

Gauged Stations
1. Azad-Pattan, 2.Nosheri, 3.Muzaffarabad, 4. Naran, 5. Chinari, 6. Garhi-Habibullah, 7. Kohala, 8. Palote, 9. Kotli, 10. Karoli, 11.Mangla

Fig. 3. Mangla basin map.

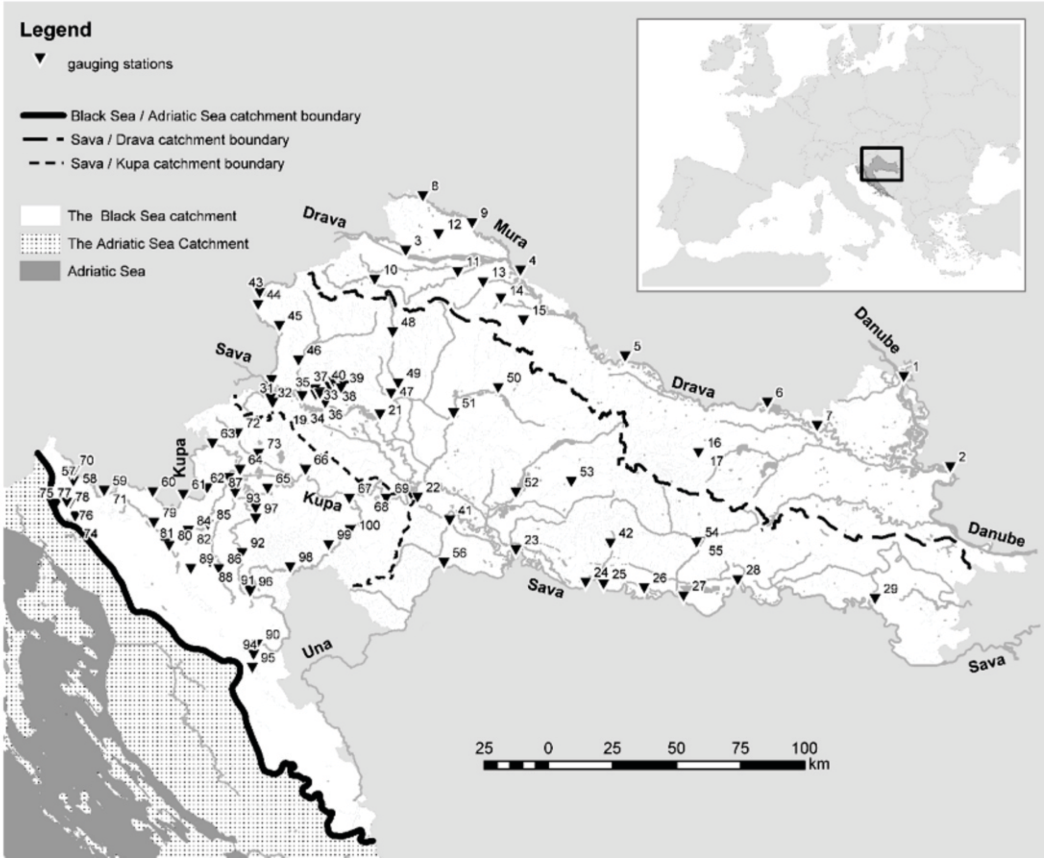


Fig.4. Map of Danube River Basin.

3. Results and discussion

3.1. Development of envelope curve (Linear Relationship between peak discharge as calculated using Creager equation Vs area of sub-catchment)

The envelope curves of both Indus and Jhelum river basins are shown in figure 5. These envelope curves show the relationship between catchment area and maximum peak discharges (as shown in table 3) observed at gauged sub-catchments of Indus and Jhelum River basins. In case of Jhelum basin envelope curve, R2 value is 0.999 showing a weak relationship between catchment area and observed peak discharge because two stations namely Talhata (2354 km^2) and Ghari-Habibullah (2382 km^2) approximately have similar catchment area but large difference between their maximum observed peak can be seen. On other hand, Palote (1111 km^2) and Naran (1036 km^2) also have approximately similar catchment area but have a large difference between their maximum observed peaks. Though, R2 relationship was not of primary concern while establishing this relationship but the focus was to draw an envelope curve. The R2, however, provides an insight into the hydrological behavior of sub-catchment with regards to their catchment area. Envelope curve for Jhelum River basin is not showing a good trend with respect to data points because the major problem in Jhelum River basin is the scarcity of flow data.

For Indus River basin envelope curve R2 value is 0.9928, showing a good relationship because the stations having similar catchment area do not possess large variations in their observed peak discharges. This figure shows that both of these envelope curves are approximately at same positions. Envelope curve of Jhelum River basins gives slightly higher discharges than Indus envelope curve. As the slopes of two envelope curves are very close, a single envelope curve can be used for both of these river basins (as shown in figure 6).

Figure 6 shows Combine envelope curve for Indus and Jhelum River basins. The envelope curve thus obtained can be used to get maximum peak discharge for any given area present in Indus and Jhelum River basins. Discharge obtained by using this envelope curve can be used as a design discharge for small projects. This envelope curve can also be used in preliminary studies and in getting quick rough estimations of flood values that can be occur in basin of certain size.

3.2. Development of Indus and Jhelum envelope curve using different Creager's coefficient values

It has already explained that creager's equation has been used worldwide with different coefficient values. Equation (2.2) was used with different C values for the development of Indus and Jhelum envelope curve. Different Creager's coefficient values were used as a trial until envelope curve covered all the flood peaks of stream gauging stations.

Figure 7 shows envelope curves with different C values. It is clear from this figure that envelope curve with C= 23 covered all the observed peaks. Whereas envelope curve with C=5 and C=12 could not cover all the observed peaks. This shows that C=23 is suitable to draw the Indus and Jhelum envelope curve.

The envelope curve obtained by best fit line approach is shown in figure 8. Lower line showing the best fit line of data points whereas upper line showing the parallel line drawn to the best fit line. Comparison between figure 6 and figure 8 show that trend of both envelope curves is same.

It can be seen from these envelope curves (Fig. 6 and 8) that for a selected catchment area of 30,000 km^2 two of these curves given 20,000 m^3/s discharge value. This comparison shows that both of these approaches can be used for the development of envelope curve in Pakistan. Best fit line approach is simple one as compare to creager's equation method and it gives approximately similar trend like creager's envelope curve.

3.2.1. Estimation of upper bound using envelope curve

Envelope curves are a traditional method to appraise the upper bound of flood event. To improve flood frequency analysis at high return periods, the upper bound was derived from figure 6 for each gauging station present in Indus and Jhelum River basins and was integrated as supplementary information in a distribution function. Four distributions, log-normal, log-Pearson type Ш, Gumbel and two parameter gamma were selected for this purpose. All of these distributions then applied on each gauge site.

For Azad Pattan station, log-Pearson type Ш distribution crossed the upper bound at 2000 return period. Log-normal, Gumbel and two parameter gamma estimate lower discharges even at 10,000 return periods as compare to upper bound. Comparison between distributions show that all the selected distributions estimate similar discharges for short return periods. But result of these distributions varies significantly for high return periods. Upper bound derived from figure 6 for Azad Pattan station is 18495 cumec. Results show that log-Pearson type Ш distribution was unbounded and it crossed the upper bound envelope curve so it is not fit for this Azad Pattan station. Results are shown in figure 9.

In case of Dhoke Pattan station, Log-normal and log-Pearson type Ш distributions crossed the upper bound. Log-normal distribution crossed the upper bound at 900 return periods and log-Pearson Ш crossed it at 5000 return period. Comparison shows that Gumbel and Two parameter gamma estimate similar discharges for lower, middle and even for high return periods. Upper bound value derived from figure 6 for Dhoke pattan station is 8900 cumec. Log-normal and log-Pearson type Ш distributions were unbounded so these distributions are not fit for Dhoke pattan station. Results are shown in figure 10.

In case of Dhoke Pattan station, Log-normal and log-Pearson type Ш distributions crossed the upper bound. Log-normal distribution crossed the upper bound at 900 return periods and log-Pearson Ш crossed it at 5000 return period. Comparison shows that Gumbel and Two parameter gamma estimate similar discharges for lower, middle and even for high return periods. Upper bound value derived from figure 6 for Dhoke pattan station is 8900 cumec. Log-normal and log-Pearson type Ш distributions were unbounded so these distributions are not fit for Dhoke pattan station. Results are shown in figure 10.

Table 3. Peak discharges along with their respective catchment area.

Stations	Peak Discharge (cumec)	Catchment Area(sq-km)
Alam bridge	4222.5	26159
Gilgit	2990.4	12095
kot murtaza	3460.0	36001
Thal	833.9	5543
Chahan	272.9	241
Chirah	1462.0	326
Dhok pattan	5412.7	6475
Guriala	3344.0	3056
Nowshera	6177.8	88578
janshi post	293.0	1847
Chakdara	7346.3	5776
kalam	485.9	2020
Chitral	1602.0	11396
Phulra	580.8	1057
Daggar	183.1	598
Karora	1133.5	635
Dhoyan	1090.0	4040
yogo	3623.0	33670
Bisham qila	37151.9	162393
Kachura	6816.0	112665
Khairabad	16726.0	252525
Kharmong	2805.0	67858
Masan	25646.4	287490
Shatial bridge	12602.0	150220
Azad Pattan	10822	26485
Chinari	1876	13598
Gari-Habibullah	1626	2382
Kohala	10199	24890
Kotli	6561	3238
Mangla	7571	33411
Muzaffar Abad	3766	7278
Naran	448	1036
Nosheri	3037	6809
Palote	3287	1111
Domel	3765	14504
Talhata	421	2354
Hattian Bala	1574	13938

Table 4.Trial with different Creager coefficient values.

Trials	Values of C	Remarks
1	5	17 points were enveloped
2	12	29 points were enveloped
3	23	All points were enveloped

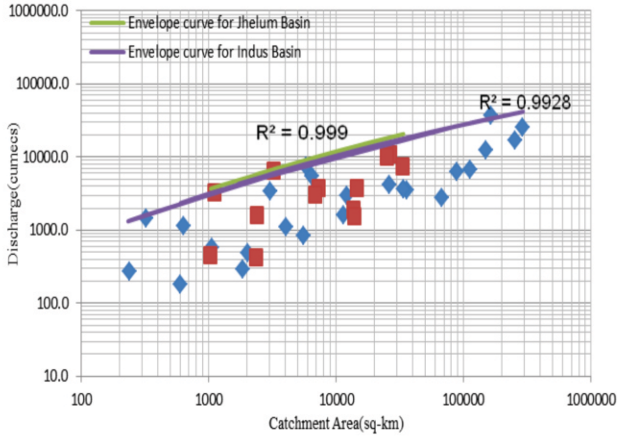


Fig. 5. Envelope curves for Indus and Jhelum river basins.

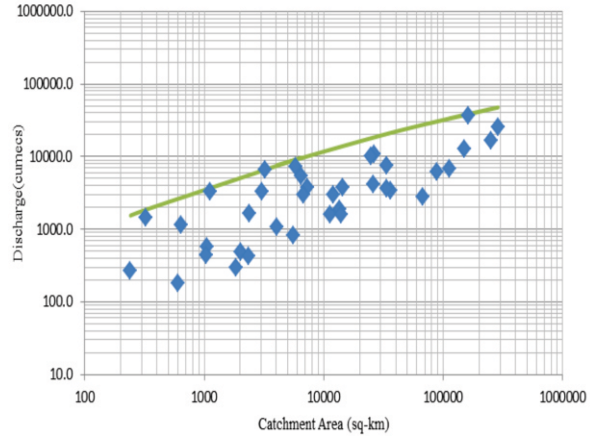


Fig. 6. Combine Envelope curve of Indus and Jhelum basin.

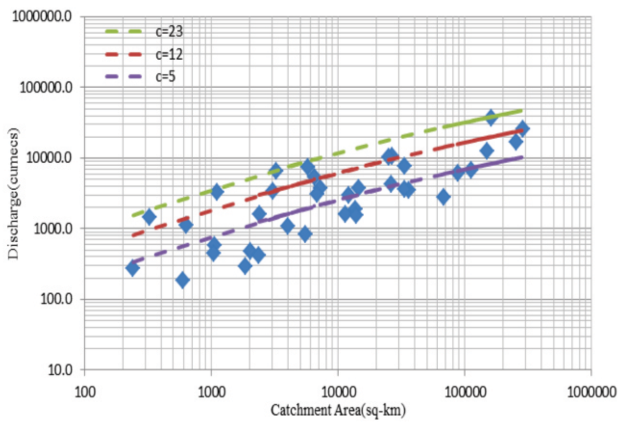


Fig. 7. Envelope curves for different C values.

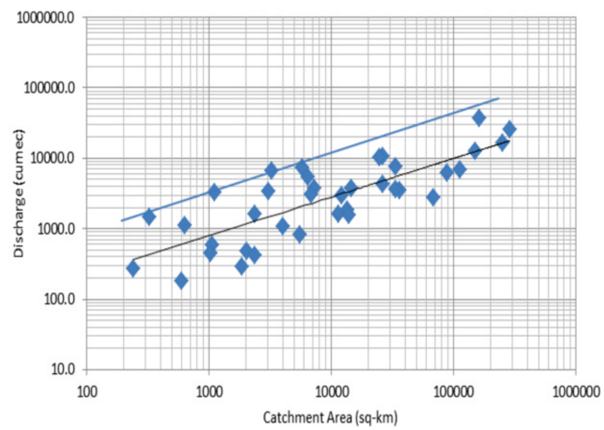


Fig. 8. Combine Envelope curve for Indus and Jhelum Basin.

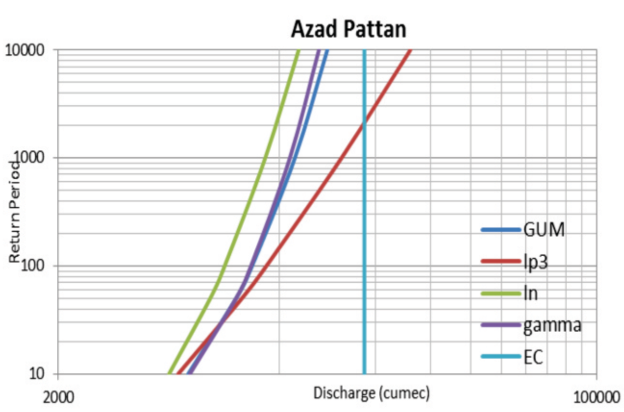


Fig. 9. Distribution results for Azad Pattan station.

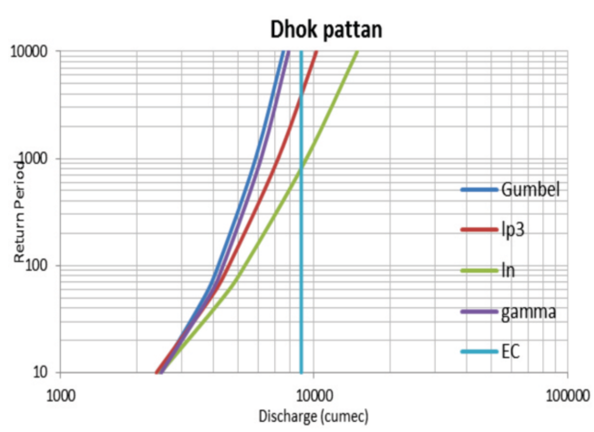


Fig. 10. Distribution results for Dhoke Pattan station.

3.2.1. Estimation of peak flood for certain sub-basin with the envelope curve and its comparison with empirical formulae

Different stations were selected for the estimation of peak flood by envelope curve. Than the estimated flood discharges were compared with the discharges estimated by Dicken's formula as shown in Table 5 .

Comparison shows that peak flood discharges estimated by Dicken's formula and envelope curve are approximately similar for those stations which have small catchment area. But for large catchment area both of these approaches show big difference between their results. The reason is that empirical formulae are not suitable for large catchment areas. These are regional formulae and usually designed for small catchments. So envelope curve is a suitable method for quick rough estimation of peak flood for large catchment areas.

3.2.2. Comparison of Indus Jhelum envelope curve with Danube River basin envelope curve

Creager and Francou-Rodier have developed envelope curves for Danube river basin. Creager envelope curve was selected for comparison purpose. Comparison between Danube river envelope curve and Indus Jhelum envelope curve is shown in figure 11.

Figure 11. shows the comparison of envelope curves developed for Danube River basin and verified combined envelope curve of Indus and Jhelum River basins developed in the present study. The comparison shows that Danube River basin envelope curve gives smaller values of maximum discharges than Indus Jhelum envelope curve for all catchment areas. It can be seen that for same catchment area at some stations, Indus Jhelum envelope curve gives higher discharges as compare to Danube River envelope curve. This is due to different hydro-meteorological and topographical features of these river Basins.

Table 5. Comparison of Peak flood estimated by different methods.

Station Name	Catchment Area	Discharge by Dicken's formula (m^3/s)	Discharge by Envelope curve (m^3/s)
Chirrah	326	1467	1462
Daggar	598	2691	2600
Karora	635	2857	2800
Phulra	1057	4756	3500
Janshi post	1847	8311	5000
Kalam	2020	9090	5500

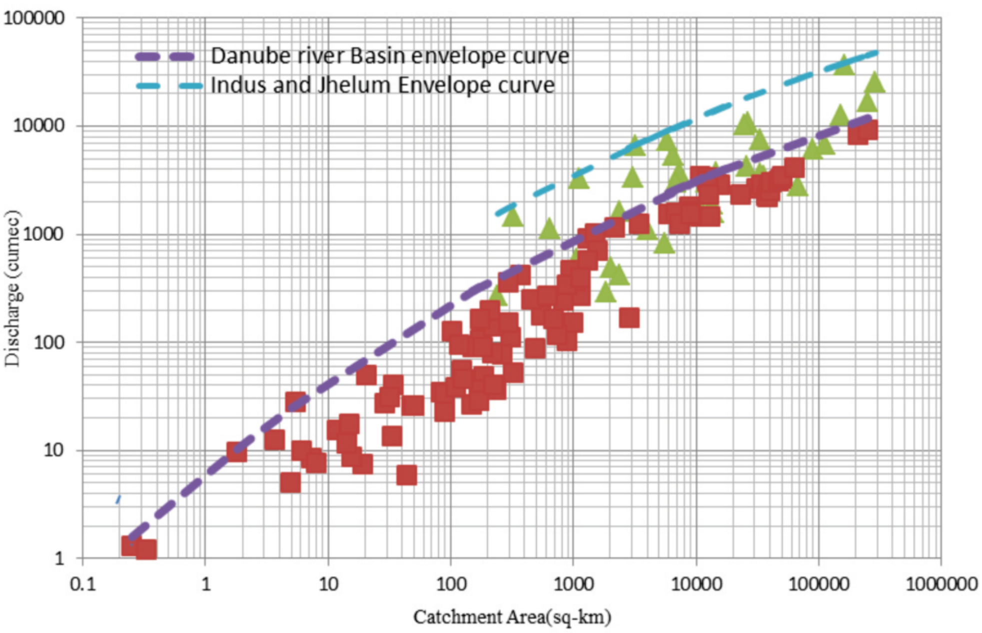


Fig. 11. Comparison of envelope curves.

4. Conclusions

Envelope curves for maximum floods of the Indus and Jhelum river basins can be used for preliminary flood estimation studies in these river basins because it seems to provide more realistic discharge estimation as compare to empirical formulae which are applicable only for small catchments. Indus Jhelum envelope curve provides an upper bound value for each gauging station, which can be used as supplementary information in a distribution function. By this approach, the estimation of discharge for higher return periods seems to provide more realistic discharge estimation for higher return periods. Log-Pearson Ш distribution crossed the upper bound most of the time whereas at some stations it gave very low values of discharges even for higher return periods as compare to other distributions. Large variations were noticed in Log-Pearson Ш results. So it is not fit for Indus and Jhelum river basins whereas Gumbel distribution shown a reasonable trend for all gauging stations. It is best fit distribution for Indus and Jhelum River basins.

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